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These last three follow at once from the fact that for the pentagon, quadrilateral, and triangle, we have three, two, and one term in the expression for the area, while the power of π is for the pentagon π^4, for the quadrilateral π^3, for the triangle π^2, in the denominator.

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.

Remark by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

No. 139 is the same as No. 130 for which I have sent a solution previously.

140. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Obtain the average area of a triangle formed by a tangent to the four—cusped hypocycloid and the coördinate axes.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let a=radius of fixed circle. Then portion of tangent intercepted by coördinate axes=a. Area of triangle=$\frac{1}{2}xy$, subject to the condition $x^2+y^2=a^2$.

$$\therefore \text{Average area}=\tfrac{1}{2}\int_0^a x\sqrt{(a^2-x^2)}dx\Big/\int_0^a dx=\tfrac{1}{6}a^2.$$

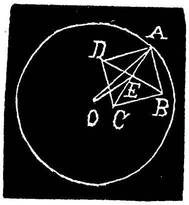
141. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Upon a circular table, radius r, a *variable* square plate is thrown at random. What is the probability that the plate will lie wholly on the table?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O be the center of the given circle, radius r; $ABCD$ the square, center E; $AB=2x$, $OE=z$, $\angle OAE=\theta$. Then $AE=x\sqrt{2}$.

If the center of the square, E, falls on a circle center O and radius $(r-x\sqrt{2})$, the square will be wholly on the table. If E falls on a circle, center O and radius z, the plate will lie wholly on the table. $z=\sqrt{[r^2+2x^2-2rx\sqrt{(2)}\cos\theta]}$. The limits of x are 0 and $\frac{1}{2}r\sqrt{a}$; of θ, 0 and $\frac{1}{4}\pi-\sin^{-1}(x/r)=\theta'$. Let p=chance. Since the whole number of ways E can fall on the circle is πr^2, we get,



$$p=\frac{\pi\int_0^{\frac{1}{2}r\sqrt{2}}[r-x\sqrt{2}]^2dx}{\pi r^2\int_0^{\frac{1}{2}r\sqrt{2}}dx}+\frac{\pi\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta}z^2dx\,d\theta}{\pi r^2\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta}dx\,d\theta}$$

$$=\frac{\sqrt{2}}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}[r-x\sqrt{2}]^2dx+\frac{[2+\sqrt{2}]}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta'}z^2dx\,d\theta$$

$$=\tfrac{1}{3}+\frac{[2+\sqrt{2}]}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}\{[r^2-2x^2][\tfrac{1}{4}\pi-\sin^{-1}\frac{x}{r}]-2x[\sqrt{(r^2-x^2)}-x]\}dx=1-\frac{\sqrt{2}}{9}.$$

MISCELLANEOUS.

136. Proposed by **W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.**

(1) Solve (to five places) the equations, $\sin(x+\frac{1}{6}\pi)=10\sin x$, and $a\cos\phi\log\sin\phi=p$ where a is small and positive, and $\varphi=\alpha+\kappa$, where κ is very small and α is not very small; (2) If $a\theta=b\varphi$ where a is prime to b, and $\sin\theta=p$, $\sin\varphi=q$, how many values of q are there for each of p ? (3) if $2x=\sin^{-1}x$, show there is only one positive value of x, and find it.

Solution by **G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

(1) $\sin(x+\frac{1}{6}\pi)=\frac{1}{2}\sqrt{3}\,\sin x+\frac{1}{2}\cos x=10\sin x$.

$\therefore(20-\sqrt{3})\sin x=\cos x$, or $\tan x=\dfrac{1}{20-\sqrt{3}}=\dfrac{20+\sqrt{3}}{397}=.05474$. $\therefore x=3°8'$.

$\cos\varphi\log\sin\varphi=p/a$ or $\log(\sin\varphi)^{\cos\phi}=p/a$.

$\therefore\sin^2\varphi=e^{2p\sec\phi/a}$.

$$1-\cos^2\varphi=1+\frac{2p}{a}\sec\varphi+\frac{4p^2}{2\,!a^2}\sec^2\varphi+\frac{8p^3}{3\,!a^3}\sec^3\varphi+\dots..$$

$$-a/2p=\sec^3\varphi+\frac{p}{a}\sec^4\varphi+\frac{2p^2}{3a^2}\sec^5\varphi+\frac{p^3}{3a^3}\sec^6\varphi+\dots..$$

By reversion of series,

$$\sec\varphi=-\left(\frac{a}{2p}\right)^{\frac{1}{2}}-\frac{1}{3}\left(\frac{p}{4a}\right)^{\frac{1}{2}}-\frac{p}{18a}-\frac{p^2}{81a^2}\left(\frac{a}{2p}\right)^{\frac{1}{2}}-\frac{29p^2}{2430a^2}\left(\frac{p}{4a}\right)^{\frac{1}{2}}\dots..$$

The value of $\sec\varphi$ to five terms. Since a is small, p is still smaller. When any multiple of $\frac{1}{2}\pi$ is substituted for φ, $\cos\varphi\log\sin\varphi$ is a maximum.

(2) $\sin\theta=\sin(m\pi-\pi)=\sin(2m\pi+\theta)=p$,

$\sin\varphi=\sin(m\pi-\varphi)=\sin(2m\pi+\varphi)=q$.

Since $a\theta=b\varphi$, but one value of q satisfies one value of p; for $a(n\pi\pm\theta)=b(n\pi\pm\varphi)$ only when $a=b$ and $a\theta=b(n\pi\pm\varphi)$ never.